

The Generalized Theory of Landau Damping and Plasma-Gravitational Analogy

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The collisionless damping of electron plasma waves was predicted by Landau in 1946 [1] and later was confirmed experimentally. Landau damping plays a significant role in many electronics experiments and belongs to the most well known phenomenon in statistical physics of ionized gases. In spite of transparent physical sense, the effect of Landau damping has continued to be of great interest to theorist as well. Much of this interest is connected with counterintuitive nature of result itself coupled with the rather abstruse mathematical nature of Landau's original derivation (including so-called Landau's rule of complex integral calculation). Moreover, for these reasons there were publications containing some controversy over the reality of the phenomenon (see for example, [2]). In paper [3] the difficulties originated by Landau's derivation were clarified. The mentioned consideration leads to another solution of Vlasov - Landau equation, these ones in agreement with data of experiments. The problem solved in this report consists in investigation of Landau damping from the positions of Generalized Boltzmann Physical Kinetics [3 - 6]. The influence of the particle collisions is taken into account. I remark here only that the generalized Boltzmann equation (GBE) describes how the one-particle distribution function f_α ($\alpha = 1, \dots, \eta$) in a η -component gas mixture changes over times of the order of the time between collisions, of the order of the hydrodynamic flow time, and, unlike the conventional Boltzmann equation, over a time of the order of the collision time. The GBE for a plasma medium has the form

$$\frac{Df_\alpha}{Dt} - \frac{D}{Dt} \left(\tau_\alpha \frac{Df_\alpha}{Dt} \right) = J_\alpha,$$

where D/Dt is the substantial derivative containing the self-consistent force, J_α is the classical (Boltzmann) collision integral,

and τ_α is the mean time between the close particle collisions. The generalized Boltzmann equation in general and that for plasma in particular have a fundamentally important feature that the additional GBE terms prove to be of the order of the Knudsen number. This does not mean that in the hydrodynamic (small Kn) limit these terms may be neglected: the Knudsen number in this case appears as a small parameter of the higher derivative in the GBE. Consequently, the additional GBE terms (as compared to the BE) are significant for any Kn , and the order of magnitude of the difference between the BE and GBE solutions is impossible to tell beforehand. In this connection, it is of interest to apply the GBE model to obtain the dispersion relation for plasma in the absence of a magnetic field. The dispersion equation for electrons in the generalized Boltzmann theory (using the assumptions that were used by Landau in the BE-based derivation and the collisional term written in the Bhatnagar - Gross - Krook (BGK) form with the relaxation time ν_{re}^{-1}) has the following form:

$$1 + \frac{1}{r_D^2 k^2} \left[1 - \frac{m_e}{\sqrt{2\pi k_B T}} \int_{-\infty}^{+\infty} \frac{[i - \tau(\omega - ku)](\omega - \nu_{re}) e^{-m_e u^2 / 2k_B T}}{i(\omega - ku) - \tau(\omega - ku)^2 - \nu_{re}} du \right] = 0,$$

where $r_D = \sqrt{\epsilon_0 k_B T / n_e e^2}$ is Debye - Hückel radius, k_B - Boltzmann constant, k is wave number. The principle of regularization of the Landau singular integral is used, which differs of the Landau rule. Namely: in the linear problem of interaction of individual electrons with waves of potential electric field the natural assumption can be introduced that solution depends only of concrete $\omega = \omega' + i\omega''$, but does not depend of another possible modes of oscillations in physical system. The exact solution of the corresponding dispersion equations is obtained. The results of calculations lead to existence of discrete spectrum of frequencies and discrete spectrum of dispersion curves. For example Figures 1 and 2 reflect the result of calculations for 200 discrete levels for the case of the large Coulomb logarithm Λ .

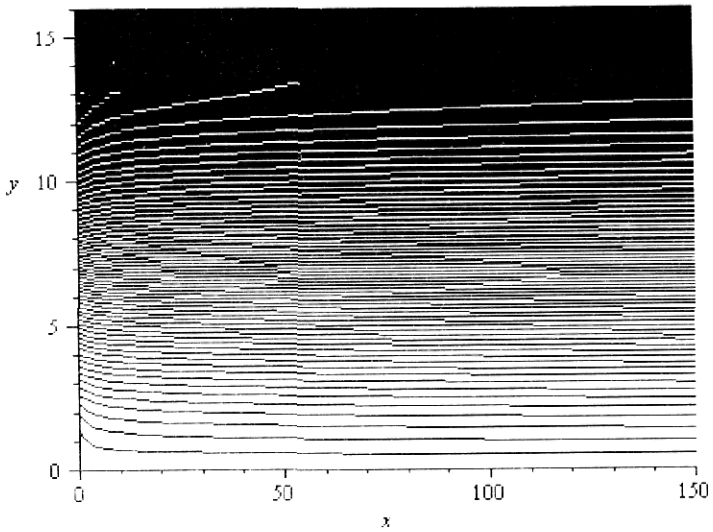


Fig.1 The dimensionless frequency $\hat{\omega}'$ (y axes) versus parameter $r_D k$ (x axes); (left)

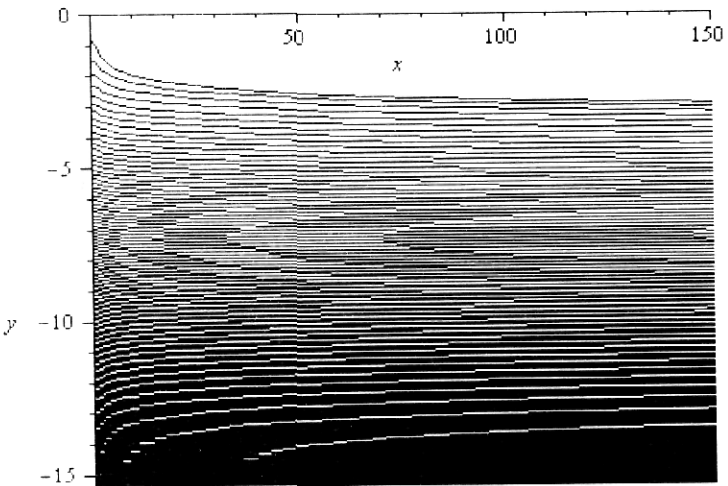


Fig. 2 The dimensionless frequency $\hat{\omega}''$ (y axes) versus parameter $r_D k$ (x axes); (right)

Plasma – gravitational analogy is well-known and frequently used effect in physical kinetics. The origin of analogy is simple and is connected with analogy between Coulomb law and Newtonian law of gravitation. From other side electrical charges can have different signs whereas there is just one kind of “gravitational charge” (i.e. masses of particles) corresponding to the force of attraction. This fact leads to the extremely important distinctions in formulation of the generalized theory of Landau damping in gravitational media, to appearance of “gravitational window” in the vicinity of parameter $r_A k = 1$ ($r_A = \sqrt{k_B T / (4\pi\gamma_N m^2 n)}$) and explanation of the Hubble effect in the frame Newton law of gravitation. The main origin of Hubble effect (including the matter expansion with acceleration) is self – catching of expanding matter by the self – consistent gravitational field in conditions of weak influence of central massive bodies.

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